Int. J. Heat Mass Transfer. Vol. 1, pp. 23–27. Pergamon Press 1960. Printed in Great Britain.

EXPERIMENTAL METHODS APPLIED TO THE DETERMINATION OF SOME TEMPERATURE RADIATION PARAMETERS

D. T. KOKOREV
Institute of Chemical Engineering, Moscow, U.S.S.R.

(*Received* 16 *November* 1959)

Abstract—The analytical determination of integral angular radiation factors is very complicated for most practical cases. The known graphical and analytical methods do not present a satisfactory solution of problem as a whole.

In this work the writer makes an attempt to solve the problem experimentally. Analytical relations were obtained, which make it possible to establish simple experimental methods of determining angular radiation factors for a general case of a spatial problem, which plays an important role in the solution of engineering problems of heat transfer by radiation. In addition, it was possible to find the optical characteristics of grey surfaces: the average factors of absorption, reflection and radiation.

Résumé—Des formules analytiques, fondées sur la théorie moderne du rayonnement thermique, ont été établies. Ces formules conduisent à des méthodes expérimentales simples permettant de déterminer le coefficient géométrique moyen de rayonnement dans le cas de problèmes très généraux et de surfaces grises.

Zusammenfassung—Auf der Grundlage der modernen Theorie der Temperaturstrahlung wurden analytische Formeln abgeleitet, mit deren Hilfe man zu einfachen experimentellen Methoden bei der Bestimmung des mittleren Flächenverhältnisses im allgemeinen Falle und bei grau strahlenden Oberflächen gelangt.

Abstract—Аналитическое определение интегральных угловых коэффициентов излучения для большинства практических случаев сопряжено с непреодолимыми трудностями. Известные графические и графоаналитические способы также не приводят к удовлетворительному решению вопроса в целом.

В данной работе предпринята попытка решить задачу экспериментальным путём. С этой целью найдены аналитические соотношения, которые позволили создать простые экспериментальные методы определения угловых коэффициентов излучения в общем случае пространственной задачи, что весьма важно для теплотехнических расчётов лучистого теплообмена. Кроме этого, оказалось возможным находить оптические характеристики серых поверхностей: интегральных коэффициентов поглощения, отражения и излучения.

THE average geometrical factor is of great importance when the modern analytical methods of heat transfer theory by radiation are being applied to the solution of engineering problems.

$$\phi_{ik} = \frac{1}{\pi F_i} \int_{F_i} \int_{F_k} \cos \theta_{ik} \,.\, d\omega_{ik} \,.\, dF_k \qquad (1)$$

This coefficient expresses physically the relation of radiant heat flow Q_{ak} emitted by a black surface i and which is incident upon the surface k, to the total radiating heat flow emitted by the surface Q_{0i} into the semispace in the direction of k.

θ_{ik} in the formula (1) is the angle between the direction of the radiation and the normal to the platform dF_k situated at the surface k; $d\omega_{ik}$ is an elementary solid angle with the opening angle at dF_i. The application of the integral formula (1) is rather complicated in practice.

The involved graphical and analytical methods of approximative evaluation of ϕ_{ik} [1–10] do not give a satisfactory solution to the problem.

The experimental method set forth below

simplifies not only the determination of ϕ_{ik} but also of some other optical parameters as well.

Let us consider an unenclosed radiating system of two arbitrarily situated thin and reciprocally concaved plates (1) and (2) shown in Fig. 1. Suppose that the internal surfaces of the plates (1) and (2) are grey ($0 < A_1, A_2 < 1$) and their external surfaces are black ($A_1^1 = A_2^1 = 1$) where A_1, A_2, A_1^1, A_2^1 are the total absorption coefficients.

It is assumed that we have a constant energy source with a continuous and uniform heat flow supply along the external surface of the plate.

If the plates (1) and (2) are placed (Fig. 1) into a medium permeable to heat rays then a stationary thermal regime will be established

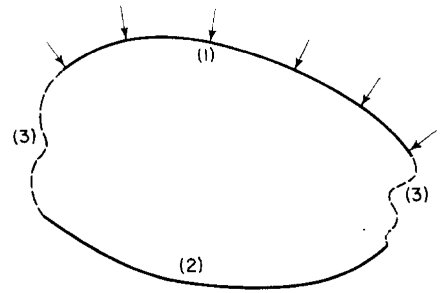

FIG. 1.

between them as a result of heat transfer by radiation and under the conditions of such a thermal regime the plates will have absolute temperatures $T_1 = \text{const.}$ and $T_2 = \text{const.}$

Using equation (1)

$$\frac{E_r(M)}{A(M)} - \int_F R(N)\frac{E_r(N)}{A(N)} K(M,N)dF_M = \int_F E_0 \,.\, K(M,N)dF_M - E_0(M), \qquad (2)$$

where E_r and E_0 are the densities of incident and black radiation, respectively, A and R grey body absorption and reflection coefficients, $K(M,N)$ dF_M an elementary geometrical factor. The indices M and N at the values entering the equation (2) indicate that these values correspond to the points at the boundary surface of the radiating body under consideration at given points M and N.

If a closed radiating system consists of a finite number n of optically uniform grey surfaces at

$$\phi_{ik} = \phi(M_i, F_k), \quad (i, k = 1, 2, 3, \ldots) \qquad (3)$$

where $\phi(M_i, F_k)$ is a radiating local geometrical factor then the equation (2) converts into a final system of linear algebraic equations of the following type [12, 13].

$$E_{ri} = A_i \sum_{k=1}^{n} A_k \,.\, \Phi_{ik} E_{ik}, \qquad (4)$$

where

$$\Phi_{ik} = \phi_{ik} + \sum_{j=1}^{n} R_j \,\Phi_{ij}\, \phi_{jk} \quad (i, k = 1, 2, 3, \ldots) \qquad (5)$$

$$E_{ik} = E_{0k} - E_{0i}$$

when E_{0k} and E_{0i} are densities of black radiation ($E_0 = \sigma_0 T^4$) relative to the surfaces k and i.

The parameter Φ_{ik} expresses the optical and geometrical properties of any pair of grey surfaces i and k depending on all the grey surfaces entering the given system. If the surfaces are black then $\Phi_{ik} = \phi_{ik}$.

The radiating system under consideration differs from those described by the equations (3) and (4) not only by the fact that this system is an unenclosed one, but also by the fact that the plate (2) is absorbing a radiating energy by one of its surfaces (internal) and radiates by both—internal and external.

The first difference is not an important one, since an unenclosed system may be easily converted into a closed one by an imaginary cover (3) (Fig. 1), the optical properties of which may depend upon the surrounding conditions. For example, in the system under consideration this cover has black body properties ($A_3 = 1$) at absolute zero ($T_3 = 0$).

The second difference is more important since it leads to a different expression for E_{r2} than follows from the equations (3) and (4).

So, taking all this into account we can find the following expression for the density of the total radiation of the plate

$$E_{r2} = A_2 \sum_{k=1}^{3} A_k \,\Phi_{2k}\, E_{k2} - E_{02} \qquad (6)$$

In the case of the steady-state thermal regime for a given radiating system $E_{r2} = 0$, and therefore equation (6) becomes

$$A_2 A_1 \Phi_{21} E_{12} + A_2^2 \Phi_{22} E_{22} + A_2 A_3 \Phi_{23} E_{32} = E_{02} \qquad (7)$$

But as we have

$$A_3 = 1,$$
$$E_{22} = E_{02} - E_{02} = 0, E_{03} = 0,$$
$$E_{32} = E_{03} - E_{02} = - E_{02},$$

then from the equation (7) we get

$$A_1 \cdot A_2 \cdot \Phi_{21} E_{12} - A_2 \Phi_{23} E_{02} = E_{02} \qquad (8)$$

The conditions of the system being closed relative to the surface (2) are determined by the expression

$$\sum_{k=1}^{3} A_k \cdot \Phi_{2k} = 1 \qquad (9)$$

The solution of the equations (8) and (9) taken together relative to Φ_{21} gives:

$$\Phi_{21} = \frac{1}{A_1 \cdot A_2} (1 + A_2 - A_2^2 \Phi_{22}) \frac{E_{02}}{E_{01}} \qquad (10)$$

or, as $E_{02} = \sigma_0 T_2^4$ and $E_{01} = \sigma_0 T_1^4$, so

$$\Phi_{21} = \frac{1}{A_1 A_2} (1 + A_2 - A_2^2 \Phi_{22}) \left(\frac{T_2}{T_1}\right)^4 \qquad (11)$$

The function Φ_{22} depends not only upon the process of radiating flow reflections from the grey surface of the plate (2) upon itself as it has a concave form, but also on the geometrical configuration and optical properties of the radiating system as a whole. This function is determined from the equations (5) for $i = 1 \cdot 2$ and $k = 2$.

$$\left.\begin{aligned} \Phi_{12} - \sum_{j=1}^{3} R_j \phi_{1j} \Phi_{j2} &= \phi_{12} \\ \Phi_{22} - \sum_{j=1}^{3} R_j \phi_{2j} \Phi_{j2} &= \phi_{22} \end{aligned}\right\} \qquad (12)$$

i.e.

$$\Phi_{22} = \frac{\gamma_2(\phi_{22} + \gamma_1 R_1 \phi_{12} \phi_{22})}{1 - \gamma_1 \gamma_2 R_1 R_2 \phi_{12} \phi_{21}} \qquad (13)$$

where

$$\gamma_1 = \frac{1}{1 - R_1 \phi_{11}}, \quad \gamma_2 = \frac{1}{1 - R_2 \phi_{22}} -$$

are the reflection coefficients of the surfaces (1) and (2), respectively.

Introducing Φ_{22} from (13) into (11) we get:

$$\Phi_{21} = \frac{1}{A_1 A_2} \left[1 + A_2 - A_2^2 \frac{\gamma_2(\phi_{22} + \gamma_1 R_1 \phi_{12} \phi_{21})}{1 - \gamma_1 \gamma_2 R_1 R_2 \phi_{12} \phi_{21}}\right] \cdot \left(\frac{T_2}{T_1}\right)^4 \qquad (14)$$

Thus, taking into account the known properties of geometrical factors, one may write

$$\Phi_{21} F_2 = \Phi_{12} F_1 \qquad (15)$$

where F_1 and F_2 are the heat transfer areas of the plates (1) and (2), and hence

$$\Phi_{12} = \frac{F_2}{F_1 A_1 A_2} \left[1 + A_2 - A_2^2 \frac{\gamma_2(\phi_{22} + \gamma_1 R_1 \phi_{12} \phi_{21})}{1 - \gamma_1 \gamma_2 R_1 R_2 \phi_{12} \phi_{21}}\right] \left(\frac{T_2}{T_1}\right)^4 \qquad (16)$$

If (1) and (2) are the plane plates, then

$$\phi_{11} = \phi_{22} = 0, \gamma_1 = \gamma_2 = 1,$$

and consequently

$$\Phi_{12} = \frac{F_2}{F_1 A_1 A_2} \left(1 + A_2 - A_2^2 \frac{R_1 \phi_{12} \phi_{21}}{1 - R_1 R_2 \phi_{12} \phi_{21}}\right) \left(\frac{T_2}{T_1}\right)^4 \qquad (17)$$

$$\Phi_{21} = \frac{1}{A_1 A_2} \left(1 + A_2 - A^2 \frac{R_1 \phi_{12} \phi_{21}}{1 - R_1 R_2 \phi_{12} \phi_{21}}\right) \left(\frac{T_2}{T_1}\right)^4 \qquad (18)$$

Suppose that the opposite internal surfaces (1) and (2) are black, i.e. $A_1 = A_2 = 1$, and so $\gamma_1 = \gamma_2 = 1$. From equations (12) and (16) we get

$$\left.\begin{aligned} \Phi_{12} = \phi_{12} &= \frac{F_2}{F_1} \left(2 - \phi_{22}\right) \left(\frac{T_2}{T_1}\right)^4 \\ \Phi_{21} = \phi_{21} &= (2 - \phi_{22}) \left(\frac{T_2}{T_1}\right)^4 \end{aligned}\right\} \qquad (19)$$

for the concave surfaces and

$$\left.\begin{aligned} \Phi_{12} = \phi_{12} &= 2 \frac{F_2}{F_1} \left(\frac{T_2}{T_1}\right)^4 \\ \Phi_{21} = \phi_{21} &= 2 \left(\frac{T_2}{T_1}\right)^4 \end{aligned}\right\} \qquad (20)$$

for the plane surfaces.

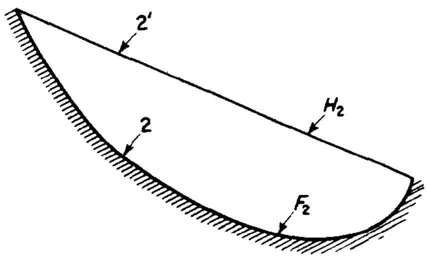

FIG. 2.

The geometrical factor ϕ_{22} characterizes the concavity of the surface (2) and may be expressed through the areas F_2 and H_2 (Fig. 2), where F_2 is the effective area of the concave surface (2), and H_2 is the area of its closing surface. From the condition of closeness

$$\left.\begin{aligned}\phi_{22} + \phi_{22'} &= 1,\\ \phi_{2'2} &= 1.\end{aligned}\right\} \tag{21}$$

But as

$$F_2\phi_{22'} = H_2\phi_{2'2} = H_2, \tag{22}$$

then

$$\phi_{22} = 1 - \frac{H_2}{F_2} \tag{21'}$$

Introducing ϕ_{22} from (21) into (19) we get:

$$\left.\begin{aligned}\phi_{12} &= \frac{H_2 + F_2}{F_1}\left(\frac{T_2}{T_1}\right)^4\\ \phi_{21} &= \left(1 + \frac{H_2}{F_2}\right)\left(\frac{T_2}{T_1}\right)^4\end{aligned}\right\} \tag{22'}$$

From the last formulae we can see that it is possible to determine ϕ_{12} and ϕ_{21} if F_1, F_2, H_2, T_1, T_2 are known. The first, the second and the third ones are usually given and it is quite possible to determine T_1 and T_2 experimentally, as shown by the writer [14].

And now using the equations (15) and (16) we can get the expressions which determine the absorption (reflection) coefficient of the grey surfaces experimentally. Let us analyse the following cases:

Assume that the inside surface of the plate (1) shown in Fig. 1 is black ($A_1 = 1$) and the outside surface of the plate (2) is grey ($0 < A_2 < 1$). If the outside surface of the plate (2) is black, then on the basis of the equation (15) the absorption coefficient A_2 becomes:

$$A_2 = \frac{T_2^4}{\phi_{21}T_1^4 - T_2^4} \tag{23}$$

The plate (2) being black from both surfaces ($A_2 = A_2^1 = 1$) and the inside surface of the plate (1) being grey ($0 < A_1 < 1$) then as it was mentioned above, we get on the basis of the equation (15)

$$A_1 = \frac{2 - \phi_{12}\phi_{21}}{\phi_{21}[(T_1/T_2)^4 - \phi_{12}]} \tag{24}$$

The internal opposite surfaces of the plates (1) and (2) are black ($A_1 = A_2 = 1$) and the outside surface of the plate (2) is grey ($0 < A_2^1 < 1$) Then the equation (6) must be rearranged to the following form:

$$E_{r2} = \sum_{k=1}^{3} A_k\Phi_{2k} \cdot E_{k2} - A_2^1E_{02} \tag{25}$$

Since

$$E_{r2} = 0,\ A_1 = A_2 = 1,$$

then

$$\Phi_{21}E_{12} - \Phi_{23}E_{02} = A_2^1E_{02} \tag{26}$$

Thus

$$\sum_{k=1}^{3} A_k\Phi_{2k} = \sum_{k=1}^{3} \phi_{2k} = \phi_{21} + \phi_{22} + \phi_{23} = 1 \tag{27}$$

or

$$\Phi_{21} = \phi_{21},\ \Phi_{23} = \phi_{23},\ \Phi_{22} = \phi_{22} = 0. \tag{28}$$

Taking this into account we get from the equation (26)

$$A_2^1 = \phi_{21}\left(\frac{T_1}{T_2}\right)^4 - 1 \tag{29}$$

In accordance with the known expressions

$$R = 1 - A \text{ and } \sigma = \sigma_0 A \tag{30}$$

where σ and σ_0 are the radiation coefficients of the grey and black bodies, respectively. The formulae (23), (24) and (29) lead to the explicit dependence

$$\sigma,\ R = f(T_1 \cdot T_2)$$

Consequently, from formulae (23), (24) and (29) it is possible to determine A, R and σ if ϕ_{12}, ϕ_{21}, T_1, T_2 are known. ϕ_{12} and ϕ_{21} may be given, and T_1, T_2 may be easily found experimentally. Therefore it is better to deal with the radiating system of two round and flat plates with the equal diameters.

It the plates are placed in parallel with the normal through their centres then (15):

$$\phi_{12} = \phi_{21} = \left[\sqrt{\left\{1 + \left(\frac{h}{d}\right)^2\right\}} - \frac{h}{d}\right]^2 \tag{31}$$

where $d = d_1 = d_2$ and h are their diameters and the distance between them, respectively.

The distance between them is taken out in accordance with the design of the apparatus.

The experimental method of determination of A, R and σ, which is described by the writer [16], was created on the basis of the previously given expressions.

The further analysis, which is not adduced here, gives the possibility of receiving the analogous expressions and the corresponding methods of the experimental determination of the radiation optical and geometrical parameters Φ_{ik}, γ_{ik} and others.

Finally we may note that the derived formulae for ϕ_{ik}, ϕ_{ki} and A are based on the assumption that the given radiating system is in an empty space of an infinite length.

However, the real condition does not correspond to this. The laboratory room where the plates (1) and (2) are placed is limited by the walls, the ceiling and the floor, and is filled with air. Because of this the surfaces of the heated plates (1) and (2) will take part not only in the convective heat transfer with air, but also in the heat transfer by radiation with floor, ceiling and the walls of the room. The above mentioned factors cannot effect the temperature of the surface (1), since the conditions T_1 = const. are provided by the heater.

On the contrary, the heat transfer by convection between the plate (2) and air, and the heat transfer by radiation of the above mentioned and other bodies, except the plate (1), lead to the fact that the temperature T_2 will turn out to be different and will not correspond to the conditions of the derivation of formulae for ϕ_{ik}, ϕ_{ki}, A, R and σ.

Nevertheless, we can choose such a temperature field for the plate (2), that the influence of the external factors on it and, consequently, on the accuracy of the determination of ϕ_{ik}, ϕ_{ki}, A becomes irrelevant.

This phenomenon is explained by the fact that the heat loss by the plate (2) because of convection at a certain temperature is compensated by the inflow of heat to it as the barriers of the room also emit radiation. For example, the investigations showed [14] that this compensation takes place at $T_2 = 323° - 333°K$ when the experiment is performed at the room conditions. T_1 and T_2 are determined by the experiment as average values on the surfaces F_1 and F_2.

REFERENCES

1. H. GRÖBER, *Die Grundgesetze der Wärmestrahlung und ihre Anwendung auf Dampfkessel mit Innenfeuerung*. Springer Verlag, Berlin (1917).
2. W. NUSSELT, *Z. Ver. dtsch. Ing.* **78**, (1928).
3. SEIBERT, *Forsch. Arb. Ingwes.* No. 324 (1930).
4. KESSLER, *Z. bayer. (Dampfk) Revis Ver.* **29**, 115 (1925).
5. H. C. HOTTEL, *Heat Transfer* by V. MACADAMS. Russian translation, Moscow (1936).
6. T. T. USENKO, *Izv. Gozn. Inst.* No. 3 (1920).
7. V. N. TIMOFEEV, *Izv. VTI*, No. 9 (1934).
8. G. L. POLYAK, *Izv. Akad. Nauk SSSR, OTN*, No. 3 (1937).
9. G. L. POLYAK, *Izv. Energet. Inst. Akad. Nauk SSSR* 3 (1937).
10. S. P. SYROMYATNIKOV, *Teplovaya Rabota Paravoznoj Topki*. Transjeldozirdat (1953).
11. U. A. SURINOV, *Izv. Akad. Nauk SSSR, OTN*, No. 7 (1948).
12. U. A. SURINOV, *Izv. Akad. Nauk SSSR, OTN*, No. 7 (1953).
13. U. A. SURINOV, *Dokl. Akad. Nauk SSSR* **33**, No. 2 (1952).
14. D. T. KOKOREV, *Trud. Mosk. inst. Khim. Mash.* **12** (1957).
15. M. V. KIRPICHEV and others, *Kotelnye Ustanovki*, Vol. 1. Energoizdat, Moscow (1947).
16. D. T. KOKOREV, *Trud. Mosk. inst. Khim. Mash.* **12** (1957).